Online Appendix for *Airport Slot Allocation Problems*

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July 29, 2022

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A The Degeneration of the LP Model

The baseline model coincides with the LP model when airlines have unit demands because in such a case, airlines' preferences are trivially lexicographic. Consider the following restrictions:

(i) no airline owns a canceled flight;

(ii) each airline owns exactly one non-canceled flight;

(iii) each airline owns at most one slot;

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(iv) no airline owns a slot;

(v) each airline owns exactly one slot;

(vi) each slot is owned by some airline.

In general, airlines' preferences are more restricted than agents' preferences in traditional allocation problems because airlines want earlier feasible slots but not arbitrary slots for their flights, while agents can have arbitrary preferences.

Under restrictions (i), (ii), and (iii), the LP model degenerates to a restricted variation of the house allocation with existing tenants problem (Abdulkadiroğlu and Sönmez, 1999), and the SMTC reduces to a variant of the top trading cycles mechanism. Under restrictions (i), (ii), and (iv), the LP model reduces to a restricted variation of the house allocation problem (Hylland and Zeckhauser, 1979), and the SMTC reduces to a variant of the serial dictatorship. Under restrictions (i), (ii), (v), and (vi), the LP model reduces to a restricted variation of the housing market (Shapley and Scarf, 1974), and the SMTC reduces to a variant of the core mechanism.[1]

B Additional Algorithms

B.1 The Alternative MTC Trading Algorithm

We introduce the *alternative MTC Trading algorithm* in this section. Let $z' : \{1, 2, ..., |E'|\} \to E'$ be an ordering. Note that z' is not defined if $E' = \emptyset$. We use $(a(\cdot), s, b(\cdot), s', ...)$ to mean $a(\cdot)$ picks s, $b(\cdot)$ picks s', etc.

[1] In a housing market (under strict preferences), there is a unique matching in the core (Roth and Postlewaite, 1977), and the Gale's top trading cycles algorithm (attributed to David Gale by Shapley and Scarf (1974)) can be used to find the outcome of the core mechanism. Restrictions (i), (ii), (v), and (vi) imply that the number of flights equals the number of slots. Since there is no vacant slot, an ordering is not needed.

Alternative MTC Trading Algorithm: According to R_a, let $a(i)$ and $a(|F_a^c| + i)$ represent the i-th most important flight in F_a^c and F_a^{uc}, respectively. Let $S^1 = S$ and $z^1 = z$.
At Step $n \geq 1$:
Let $a(\cdot)$ be the first flight and $u(\cdot)$ be the first un-inserted flight in z^n. For $v \in A$, let $v(\cdot)$ be the first flight of v in z^n and $\widehat{v}$ indicate $v \neq a$. Let each flight in F^c pick the earliest feasible slot in $S^n \cap S^c$, and each flight in F^{uc} pick the slot that it is removed with in the M-IA.
If $a(\cdot)$ picks a slot in $S \cap S_{\widehat{v}}$, modify z^n by inserting $\widehat{v}(\cdot)$ in front of $a(\cdot)$.
If $a(\cdot)$ picks a slot $s_v \in S^n \cap S_v$, then there is a cycle $(a(\cdot), s_v, v(\cdot), ..., s_b, b(\cdot), s_a)$.
If $a(\cdot)$ picks a slot $s \in S^n \cap (S_{-A} \cup S_{\cancel{A^n}})$, then there is a chain $(a(\cdot), s, u(\cdot), ..., s_b, b(\cdot), s_a)$.
Remove all flights in the cycle or chain by assigning them the slots they pick. If there is no more flight, stop. Otherwise, denote the resultant set of slots and ordering by S^{n+1} and z^{n+1}, respectively; go to the next step.

The smallest cycle is $(a(\cdot), s_a)$, where $a(\cdot) = v(\cdot)$ and $s_v = s_a$. The shortest chain is $(a(\cdot), s)$, where $a(\cdot) = u(\cdot)$.

Theorem B.1: For any z, the MTC Trading algorithm and the alternative MTC Trading algorithm produce the same outcome.

Proof of Theorem B.1: Observe that for $a \in A$ and $i \leq |F_a|$, $a(i)$ represents the same flight in both algorithms. For any set of slot $S' \subseteq S$ and any set of flights $F' \subseteq F$, the alternative MTC Trading algorithm assigns slots either through a cycle $(a(\cdot), s_v, v(\cdot), ..., s_b, b(\cdot), s_a)$ or a chain $(a(\cdot), s, u(\cdot), ..., s_b, b(\cdot), s_a)$. Let z' be the current ordering in the Alternative MTC Trading Algorithm.

For any $w \in A$, let $w(\cdot)$ be the first flight of w in z'. Observe that $w(\cdot)$ is the flight in $F' \cap F_w$ that has the highest priority in z. Therefore, $(a(\cdot), s_v, v(\cdot), ..., s_b, b(\cdot), s_a)$ is a cycle in the MTC Trading algorithm for (S', F').

$u(\cdot)$ is the first un-inserted flight in z'. By selection, $u(\cdot)$ is the first flight in some z'' with no inserted flights. Let E'' be the codomain of z'' and F'' be the set of flights that are represented by the surrogates in E''. Observe that $u(\cdot)$ is the flight in F'' that has the highest priority in z. Since $F' \subseteq F''$, $u(\cdot)$ is the flight in F' that has the highest priority in z. Hence, $(a(\cdot), s, u(\cdot), ..., s_b, b(\cdot), s_a)$ is a cycle in the MTC algorithm for (S', F').

For any (S', F'), the alternative MTC Trading algorithm finds and removes a cycle in the MTC Trading algorithm. In the MTC algorithm, if a cycle is not removed at some step, then it would still be a cycle at the next step. These two facts imply that for any z, the MTC algorithm and the alternative MTC Trading algorithm produce the same outcome. ■

Example 9 (continues): Now we run the Alternative MTC trading algorithm. As before, $a(1), a(2), a(3)$, and $a(4)$ represent $f_{a,3}, f_{a,1}, f_{a,2}$, and $f_{a,4}$, respectively. $b(1)$ represents $f_{b,1}$

and $c(1)$ represents $f_{c,1}$. $S^1 = S_1$ and $z^1 = z$. In step 1, $a(1)$ picks $s_4 \in S^1 \cap S_c$, so $c(1)$ is inserted in front of $a(1)$. $z^2 = c(1), a(1), a(2), b(1), a(3), a(4)$. In step 2, $c(1)$ picks $s_5 \in S^2 \cap S_a$ and $(a(1), s_4, c(1), s_5)$ is a cycle. $z^3 = a(2), b(1), a(3), a(4)$. In step 3, $a(2)$ picks $s_1 \in S^3 \cap (S_{-A} \cup S_{\cancel{A}3})$ and $(a(2), s_2)$ is a chain. $z^4 = b(1), a(3), a(4)$. In step 4, $b(1)$ pick $s_6 \in S^4 \cap S_a$, so $a(3)$ is inserted in front of $b(1)$. $z^5 = a(3), b(1), a(4)$. In step 5, $a(3)$ picks $s_1 \in S^5 \cap (S_{-A} \cup S_{\cancel{A}5})$ and $b(1)$ is the first un-inserted flight in z^5, so $(a(3), s_1, b(1), s_6)$ is a chain. $z^6 = a(4)$. In step 6, $a(4)$ picks $s_3 \in S^6 \cap (S_{-A} \cup S_{\cancel{A}6})$ and $(a(4), s_3)$ is a cycle. The Alternative MTC trading algorithm stops as there is no more flight.

B.2 A Class of Identification Algorithms

Consider an algorithm that replaces $f_\dagger$ in the M-IA by $f_\ddagger$, which is a flight in $F_t \cap F_{[e_f, s_\circ]} \cap F_a$ that is arbitrarily selected or selected by some rules. Let Π_t^m be a landing schedule in such an identification algorithm. S_t^m and F_t^m are defined accordingly.

Theorem B.2: For $t \geq 1$, s satisfies (E) at Π_t if and only if s satisfies (E) at Π_t^m, and it satisfies (L) at Π_t if and only if it satisfies (L) at Π_t^m.

This result parallels Theorem 3. The proofs of Lemma 5, Lemma 6, and Theorem 3 do not rely on how the flight in $F_t \cap F_{[e_f, s_\circ]} \cap F_a$ is chosen, which means parallel proofs of them can show Π^m also has the same properties. So the proof of Theorem B.2 is omitted. The counterparts of Corollary 5 to 11 are also straightforward.

C Additional Details

C.1 The Performance of Compression in the Lexicographic Preference Domain

F	$f_{a,1}$	$f_{a,2}$	$f_{b,1}$
e	2	1	1
R	1	2	1
e'	2	3	1

S	s_1	s_2	s_3
Π	$f_{a,2}$	$f_{b,1}$	$f_{a,1}$
Π'	$f_{b,1}$	$f_{a,1}$	$f_{a,2}$

Suppose Π is the default landing schedule. If e is reported, Compression outputs Π, which is not Pareto efficient and not in the core as both a and b prefer Π' to Π. If a reports $e'_{f_{a,1}} = 2$ and $e'_{f_{a,2}} = 3$, s_1 becomes vacant. Compression then outputs Π', so it is not strategy-proof.

C.2 Manipulable by Postponing a Flight Cancellation

A schedule mechanism φ is **manipulable by postponing a flight cancellation** if there is an instance I, $a \in A$, and $s \in S_a$ such that $\Pi_a^{\Phi^{\varphi(I_{\to S\setminus\{s\},\Phi_\dagger^{a-s}})}(a)\cup\{s\},\star} \succ_a \varphi^\star_a(I)$.[2] Example 8 also shows φ^Z is manipulable by postponing a flight cancellation.

D A Class of Lottery Mechanisms

The main goal of this section is to propose a carefully designed lottery mechanism, ϕ^M. Examples are relegated to Section D.1. A supplementary algorithm is provide for the mechanism in Section D.2.

We begin with introducing some new concepts. For each $a \in A$, let o_a be the number of originally scheduled flights of a and k_a be the number of frozen flights of a. Flights might be canceled in the GDP or before the GDP starts. For each $a \in A$, let m_a be the number of canceled flights of a. The set of flights owned by a, F_a, to be more specific, is the set of non-canceled and non-frozen flights of a. We assume $o_a = k_a + m_a + |F_a|$. Let $n_a = m_a + |F_a|$ and $n = (n_a)_{a\in A}$. Now we extend an instance in the LP model by including n, that is, $I = (A, F_\dagger, S, \Phi_\dagger, e, R, n)$.

Let $\mathcal{M}$ be the set of landing schedules. A **schedule lottery** $\mathcal{L}$ is a probability distribution over $\mathcal{M}$. Let $\Delta\mathcal{M}$ denote the set of schedule lotteries. We denote a schedule lottery by $\mathcal{L} = \sum p_\Pi \cdot \Pi$ where $p_\Pi \in [0,1]$ is the probability weight of landing schedule Π and $\sum_\Pi p_\Pi = 1$. To extend an airline's preference to schedule lotteries, we assume an airline only cares about the expected delays of its flights. Given a schedule lottery $\mathcal{L} \in \Delta\mathcal{M}$, the **expected delay** for f is $d_f(\mathcal{L}) = \sum_\Pi p_\Pi \cdot d_f(\Pi)$. For any schedule lotteries $\mathcal{L}$ and $\mathcal{L}'$, $\mathcal{L} \succ_a \mathcal{L}'$ if and only if the first non-zero coordinate of $d_a = (d_1, d_2, ..., d_{|F_a|})$ is positive, where $d_i = d_{R_a(i)}(\mathcal{L}') - d_{R_a(i)}(\mathcal{L})$ for $i \in \{1, ..., |F_a|\}$; other cases are similar as before. A **schedule lottery for** a, $\mathcal{L}_a$, is a probability distribution over the set of landing schedules for a. A schedule lottery is a probability distribution over landing schedules, and each landing schedule Π induces a landing schedule for a, Π_a, for each $a \in A$; therefore, a schedule lottery, $\mathcal{L}$, induces a schedule lottery for a, $\mathcal{L}_a$, for each $a \in A$. We also use $\succsim_a$ to compare landing schedules for a. An airline only cares about its own flights, so for each $a \in A$, $\mathcal{L} \succsim_a \mathcal{L}' \iff \mathcal{L}_a \succsim_a \mathcal{L}'_a$.

[2] Our definition is slightly different from the one in SA because self-optimization is not assumed in their definition. Indeed, in our language, their definition would be satisfied if there exist a $\Pi_a^{\Phi^{\varphi(I_{\to S\setminus\{s\}})}(a)\cup\{s\}}$ that is better than the $\varphi_a(I)$. However, it is easy to see that if $\Pi_a^{\Phi^{\varphi(I_{\to S\setminus\{s\}})}(a)\cup\{s\},\star}$ is better than $\varphi_a(I)$, then the requirement is satisfied, and if $\Pi_a^{\Phi^{\varphi(I_{\to S\setminus\{s\}})}(a)\cup\{s\},\star}$ is not better than $\varphi_a(I)$, then no other $\Pi_a^{\Phi^{\varphi(I_{\to S\setminus\{s\}})}(a)\cup\{s\}}$ is better than $\varphi_a(I)$.

A *(direct) lottery mechanism* selects a schedule lottery $\phi(I)$ for each instance I.[3] Let $\phi_a(I)$ be the schedule lottery for a induced by $\phi(I)$. Given a schedule lottery $\phi(I)$, let $\phi^i(I)$ be a realization of $\phi(I)$. Let $\phi^i_f(I)$ be the slot that is assigned to f at $\phi^i(I)$. A lottery mechanism is *ex post core-selecting* if for any instance, it only gives positive probabilities to landing schedules that are in the core. A lottery mechanism is *ex post feasible*, *ex post non-wasteful*, *ex post individually rational, and ex post Pareto efficient* if for any instance, it only gives positive probabilities to landing schedules that are feasible, non-wasteful, individually rational, and Pareto efficient, respectively.

A lottery mechanism is *strategy-proof* if truth-telling is a dominant strategy in its induced preference revelation game. Given a landing schedule $\phi^i(I)$, let $\Phi^{\phi^i(I)}$ be the induced slot ownership function. Let $\phi^\star_a(I) = \sum p_{\phi^i(I)} \cdot \Pi_a^{\Phi^{\phi^i(I)}(a),\star}$. For $f \in F_a$, $\Pi_a^{\Phi^{\phi^i(I)}(a),\star}(f)$ is the slot that is assigned to f at $\Pi_a^{\Phi^{\phi^i(I)}(a),\star}$. Given $\phi(I)$, $\phi^\star_a(I)$ is the schedule lottery for a that a uses its slots optimally at every realization of $\phi(I)$. We call $\phi^\star_a(I)$ the *derived schedule lottery of* $\phi_a(I)$. When airline a compares $\phi_a(I)$ and $\phi_a(I')$, it does no compare them directly but compare their derived schedule lotteries, $\phi^\star_a(I)$ and $\phi^\star_a(I')$. In other words, even if $\phi_a(I') \succ_a \phi_a(I)$, as long as $\phi^\star_a(I) \succ_a \phi^\star_a(I')$, a would choose $\phi_a(I)$ over $\phi_a(I')$. A lottery mechanism ϕ is *strategy-proof* if for any I, any $a \in A$, $\phi^\star_a(I) \succsim_a \phi^\star_a(I_{\rightarrow e^a, R^a})$.

For any instance I, let $I_{\rightarrow S', \Phi'_\dagger, n'}$ denote the instance that is the same as I except with $S, \Phi_\dagger$ and n replaced by $S', \Phi'_\dagger$ and n', respectively. Let n^{a-1} be the resultant profile after n_a in n is replaced by $n_a - 1$. A lottery mechanism ϕ is *manipulable via slot destruction* if there is an instance I, $a \in A$, and $s \in S_a$ such that $\phi^\star_a(I_{\rightarrow S\setminus\{s\}, \Phi^{a-s}_\dagger, n^{a-1}}) \succ_a \phi^\star_a(I)$. A lottery mechanism ϕ is *manipulable by postponing a flight cancellation* if there is an instance I, $a \in A$, and $s \in S_a$ such that $\mathcal{L} \succ_a \phi^\star_a(I)$, where $\mathcal{L} = \sum p_{\phi^i(I_{\rightarrow S\setminus\{s\}, \Phi^{a-s}_\dagger, n^{a-1}})} \cdot \Pi_a^{\Phi^{\phi^i(I_{\rightarrow S\setminus\{s\}, \Phi^{a-s}_\dagger, n^{a-1}})}(a)\cup\{s\},\star}$.

We define a class of lottery mechanisms: a *multiple trading cycles mechanism with random ordering process (MTCR)* is a lottery mechanism that selects a schedule lottery for each I using a *random ordering process* and the MTC algorithm:

Random Ordering Process: Randomly select an ordering z^{EA} from a given distribution over $Z^E \cap Z^A$ and call it z.

MTC Algorithm: As in the main text.

Corollary C.1: Any MTCR is ex post feasible, ex post non-wasteful, ex post individually rational, ex post Pareto efficient, and ex post core-selecting.

This corollary is immediate from Proposition 3 and Theorem 4. We consider an MTCR is more suitable than an MTC in many situations.[4] We define the *multiple trading cycles*

[3]Given any I, a schedule mechanism φ can be viewed as a lottery mechanism that selects a schedule lottery that assigns probability 1 to $\varphi(I)$.

[4]Consider a simple example: $|F_a| = |F_b| = 1$. $z^1 = a(1), b(1)$ and $z^2 = b(1), a(1)$. Suppose $S = S_{-A}$

mechanism with random ordering algorithm, ϕ^M, to be a lottery mechanism that selects a schedule lottery for each I using the random ordering algorithm and the MTC algorithm.

Random Ordering Algorithm: Create n_a copies of a for each $a \in A$. Draw a copy at a time without replacement. Denote the first copy of a by $a(1)$, the second copy of a by $a(2)$, and so on. Denote the resultant ordering by $z^\star$. For each a, eliminate each $a(i)$ with $i > |F_a|$ from $z^\star$ and denote the resultant ordering by z.
MTC Algorithm: As in the main text.

It is clear that when $n_a = |F_a|$ for each $a \in A$, $z^\star = z$. Consider an example where $n_a = 2 > |F_a| = 1$ and $n_b = |F_b| = 1$. Let $z^1 = a(1), a(2), b(1)$, $z^2 = a(1), b(1), a(2)$, and $z^3 = b(1), a(1), a(2)$. Each of them realizes with probability $\frac{1}{3}$.[5] Suppose $z^\star = a(1), a(2), b(1)$. Then $z = a(1), b(1)$ and $z \in Z^E \cap Z^A$.

Since $|E|$ is finite, $|Z^E|$ is finite. So $|Z^E \cap Z^A|$ is also finite. Observe that the random ordering algorithm selects an ordering in $Z^E \cap Z^A$, so $\sum p_{z^{EA}} \cdot z^{EA} = 1$, where $p_{z^{EA}}$ is the probability that z^{EA} is realized; in addition, for each $z^{EA} \in Z^E \cap Z^A$, $p_{z^{EA}} > 0$.[6] This means the random ordering algorithm indeed selects an ordering z^{EA} from a given distribution over $Z^E \cap Z^A$, and so it is a random ordering process. ϕ^M is an MTCR, so it has the properties stated in Corollary C.1.

We define the *multiple trading cycles mechanism with simple random ordering algorithm*, ϕ^S, to be a lottery mechanism that selects a schedule lottery for each I using the random ordering algorithm and the MTC algorithm:

Simple Random Ordering Algorithm: Create $|F_a|$ copies of a for each $a \in A$. Draw a copy at a time without replacement. Denote the first copy of a by $a(1)$, the second copy of a by $a(2)$, and so on. Denote the resultant ordering by z.
MTC Algorithm: As in the main text.

Observe that if ϕ^S is employed and $m_a > 0$, then a might have incentives to hide its cancellations in order to raise the probabilities of getting earlier positions in the resultant ordering, which might in turn reduce its expected delays. ϕ^M eliminates this type of incentives by creating n_a copies of a for each $a \in A$ (Example D.1). In some cases, by creating N_a copies of a for each $a \in A$, ϕ^M also provides incentives for airlines to report their cancellations timely (Example D.2).

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and $e_{f_a} = e_{f_b}$. If a and b are the same in every aspect, then there is no good reason to choose z^1 over z^2 deterministically and vice versa.

[5] For z^1 and z^2, $a(1)$ is drawn with probability $\frac{2}{3}$; next, $a(2)$ or $b(1)$ is drawn with probability $\frac{1}{2}$; lastly, the last surrogate is drawn with probability 1. For z^3, $b(1)$ is drawn with probability $\frac{1}{3}$; next, $a(1)$ is drawn with probability 1; lastly, $a(2)$ is drawn with probability 1.

[6] For each z^{EA}, there is at least one ordering $z^\star$ such that $z^{EA}(j) = z^\star(j)$ for $j \leq |E|$. After the elimination of $a(i)$ with $i > |F_a|$ for each a from such an $z^\star$, the resultant ordering is z^{EA}. For example, if $n_a = 2$, $n_b = n_c = 1$, and z^{EA} is $a(1), b(1)$, then the $z^\star$ with $z^{EA}(j) = z^\star(j)$ for $j \leq |E|$ could be $a(1), b(1), a(2), c(1)$ or $a(1), b(1), c(1), a(2)$.

Consider a mechanism, such as ϕ^M, that assigns $|F_a|$ slots to each $a \in A$. When $m_a > 0$, a might have incentives to hide its cancellations in order to obtain more slots, which might be useful in the subsequent instances (Example D.3). This type of incentives might be eliminated by allocating an additional m_a slots to each $a \in A$. Indeed, assigning n_a slots to each $a \in A$ is also consistent with the current procedure.[7] There are many ways to do so if preferences are defined solely on landing schedules. In Section D.2, we suggest a supplementary algorithm for ϕ^M to assign an additional m_a slots to each $a \in A$.

Similary to φ^Z, ϕ^M is manipulable via slot destruction (Example D.4) and not strategy-proof (Example D.5). Note that the concept of manipulable via slot destruction relates to freezing a canceled flight. A natural question then arises: Whether an airline can be better off by freezing a non-canceled flight $f \in F_a$ in a slot $s \in S_a$ if ϕ^M is employed?[8] The answer is maybe: Example D.6 shows this could be the case, while Example D.7 shows otherwise.

D.1 Examples

Example D.1:

F	$f_{a,1}$	$f_{a,2}$	$f_{b,1}$
e	1	1	1
R	1	2	1

Suppose a has a canceled flight $f_{a,3}$. $S = \{s_1, s_2, s_3, s_4, ...\}$. Suppose ϕ^S is employed and a reports the cancellation of $f_{a,3}$ timely. Let $z^1 = a(1), a(2), b(1)$, $z^2 = a(1), b(1), a(2)$, and $z^3 = b(1), a(1), a(2)$. Each of them realizes with probability $\frac{1}{3}$. Observe that $f_{a,1}$ gets s_1 given z^1 or z^2 and gets s_2 given z^3, so the expected delay of $f_{a,1}$ is $\frac{1}{3} \times 1$ (unit of time) $= \frac{1}{3}$; $f_{a,2}$ gets s_2 given z^1 and gets s_3 given z^2 or z^3, so the expected delay of $f_{a,2}$ is $\frac{1}{3} \times 1 + \frac{2}{3} \times 2 = \frac{5}{3}$.

Now suppose a does not report the cancellation of $f_{a,3}$. Let $z^4 = a(1), a(2), a(3), b(1)$, $z^5 = a(1), a(2), b(1), a(3)$, $z^6 = a(1), b(1), a(2), a(3)$, and $z^7 = b(1), a(1), a(2), a(3)$. Each of them realizes with probability $\frac{1}{4}$. Observe that $f_{a,1}$ gets s_1 given z^4, z^5 or z^6 and gets s_2 given z^7, so the expected delay of $f_{a,1}$ reduces to $\frac{1}{4}$; $f_{a,2}$ gets s_2 given z^4or z^5 and gets s_3 given z^6 or z^7, so the expected delay of $f_{a,2}$ reduces to $\frac{1}{2}$.

To sum up, when ϕ^S is employed, a has incentives to hide its cancellation in order to raise the probabilities of getting earlier positions in the resultant ordering, which in turn reduces its expected delays.

[7] Recall that $o_a = k_a + m_a + |F_a|$. RBS assigns o_a slots to $a \in A$. Airlines can exchange their slots via Compression or choose to freeze their flights in their slots, so each $a \in A$ possesses o_a slots at the end of a reassignment. Airline a freezes k_a of its flights implies a keeps k_a slots from reassignment. Therefore, a would possess o_a slot if it is assigned $m_a + |F_a|$ slots.

[8] Note that answering this question for φ^Z is impossible unless we know Z.

Example D.2:

F	$f_{a,1}$	$f_{b,1}$	$f_{c,1}$
e	3	3	1
R	1	1	1

Suppose $f_{a,2}$ is a canceled flight of a and has been frozen in s_1.[9] $S = \{s_2, s_3, s_4, ...\}$. Suppose ϕ^S or ϕ^M is employed and a does not report the cancellation of $f_{a,2}$. Let $z^1 = a(1), b(1), c(1)$, $z^2 = a(1), c(1), b(1)$, $z^3 = c(1), a(1), b(1)$, $z^4 = c(1), b(1), a(1)$, $z^5 = b(1), a(1), c(1)$, and $z^6 = b(1), c(1), a(1)$. Each of them realizes with probability $\frac{1}{6}$. Observe that, in both cases, $f_{a,1}$ gets s_3 given z^1, z^2 or z^3 and gets s_4 otherwise, so the expected delay of $f_{a,1}$ is $\frac{1}{2}$.

Now suppose a reports the cancellation of $f_{a,2}$ timely and thus releases s_1. Observe that the release of s_1 benefits c. If ϕ^S is employed, the expected delay of $f_{a,1}$ does not change. If ϕ^M is employed, an extra surrogate of a would be created. Let $z^1 = a(1), a(2), b(1), c(1)$, $z^2 = a(1), b(1), a(2), c(1)$, $z^3 = a(1), b(1), c(1), a(2)$, $z^4 = a(1), a(2), c(1), b(1)$, $z^5 = a(1), c(1), a(2), b(1)$, $z^6 = a(1), c(1), b(1), a(2)$, $z^7 = c(1), a(1), a(2), b(1)$, $z^8 = c(1), a(1), b(1), a(2)$, $z^9 = c(1), b(1), a(1), a(2)$, $z^{10} = b(1), a(1), a(2), c(1)$, $z^{11} = b(1), a(1), c(1), a(2)$, $z^{12} = b(1), c(1), a(1), a(2)$. Each of them realizes with probability $\frac{1}{12}$. Observe that $f_{a,1}$ gets s_4 given z^9, z^{10},z^{11} or z^{12} and gets s_3 otherwise, so the expected delay of $f_{a,1}$ is $\frac{1}{3}$.

To sum up, when a cancels a flight that was frozen in a slot s and s cannot be used by another flight of a, ϕ^S provides no incentive for a to report this cancellation, while ϕ^M provides incentives for a to report this cancellation timely.

Example D.3:

F	$f_{a,1}$	$f_{b,1}$
e	2	2
R	1	1

Suppose $s_1 \in S$ and a has a canceled flight $f_{a,2}$. Suppose a reports $e'_a = (e'_{f_{a,1}} = 2, e'_{f_{a,2}} = 1)$ and $R'_a = (R'_a(1) = f_{a,1}, R'_a(2) = f_{a,2})$; b reports truthfully. ϕ^M would assign s_1 to a. Suppose in the next instance, $e_{f_{a,1}} = e_{f_{b,1}} = 1$. Now $f_{a,1}$ gets s_1 with certainty. By contrast, if a did not hide the cancellation of $f_{a,2}$, then $f_{a,1}$ only gets s_1 with probability $\frac{1}{2}$.

Example D.4 (Example 8 Revisit):

[9] s_1 is not in S_a because the slot ownership function is defined on S.

F	$f_{a,1}$	$f_{b,1}$	$f_{b,2}$	$f_{c,1}$
e	1	3	1	4
R	1	1	2	1

S	s_1	s_2	s_3	s_4	s_5
Π^1	$f_{a,1}$	$f_{b,2}$	$f_{b,1}$	$f_{c,1}$	
Π^2	$f_{b,2}$	$f_{a,1}$	$f_{b,1}$	$f_{c,1}$	
Π^3	$f_{a,1}$	$f_{b,2}$	-	$f_{c,1}$	$f_{b,1}$
Π^4	$f_{a,1}$	$f_{b,2}$	-	$f_{b,1}$	$f_{c,1}$
Π^5	$f_{b,2}$	$f_{a,1}$	-	$f_{b,1}$	$f_{c,1}$
Π^6	$f_{b,2}$	$f_{a,1}$	-	$f_{c,1}$	$f_{b,1}$

Suppose ϕ^M is employed. Suppose $S_a = \{s_3\}$ and a has a canceled flight $f_{a,2}$. Let I be the instance where a reports e_a and R_a. Let $I_{\to S\setminus\{s_3\},\Phi_\dagger^{a-s_3},n^{a-1}}$ be the instance where a freezes $f_{a,2}$ in s_3. Note that $S^{uc} = \{s_3, s_4\}$ at I but $S^{uc} = \emptyset$ at $I_{\to S\setminus\{s_3\},\Phi_\dagger^{a-s_3},n^{a-1}}$.

In $\phi^M(I)$, $b(1)$ represents $f_{b,2}$ and $b(2)$ represents $f_{b,1}$. Whenever $b(1)$ is drawn before $a(1)$, $f_{a,1}$ obtains s_2 as in Π^2. This happens with orderings $b(1), ...$ and $c(1), b(1), ...$ The probability of getting these orderings is $\frac{2}{5} + \frac{1}{5} \times \frac{2}{4} = \frac{1}{2}$. In other words, the expected delay of $f_{a,1}$ is $\frac{1}{2}$.

In $\phi^M(I_{\to S\setminus\{s_3\},\Phi_\dagger^{a-s_3},N^{a-1}})$, $b(1)$ represents $f_{b,1}$ and $b(2)$ represents $f_{b,2}$. Only when $b(1)$ and $b(2)$ are drawn before $a(1)$, $f_{a,1}$ obtains s_2 as in Π^5 and Π^6. This happens with orderings $b(1), b(2)...$, $b(1), c(1), b(2), a(1)$, and $c(1), b(1), b(2), a(1)$. The probability of getting these orderings is $\frac{2}{4} \times \frac{1}{3} + \frac{2}{4} \times \frac{1}{3} \times \frac{1}{2} + \frac{1}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$. In other words, the expected delay of $f_{a,1}$ is $\frac{1}{3}$. Therefore, ϕ^M is manipulable via slot destruction and thus manipulable by postponing a flight cancellation.

Example D.5 (Example 7 Revisit):

F	$f_{a,1}$	$f_{a,2}$	$f_{b,1}$	$f_{b,2}$
e	1	3	4	1
R	1	2	1	2
e^a	1	4	4	1

S	s_1	s_2	s_3	s_4	s_5
Π^1	$f_{a,1}$	$f_{b,2}$	$f_{a,2}$	$f_{b,1}$	
Π^2	$f_{b,2}$	$f_{a,1}$	$f_{a,2}$	$f_{b,1}$	
Π^3	$f_{a,1}$	$f_{b,2}$		$f_{a,2}$	$f_{b,1}$
Π^4	$f_{a,1}$	$f_{b,2}$		$f_{b,1}$	$f_{a,2}$
Π^5	$f_{b,2}$	$f_{a,1}$		$f_{b,1}$	$f_{a,2}$

Suppose ϕ^M is employed. Let $I_{\to_{e^a,R^a}}$ be the instance where a reports $e'_a = (e'_{f_{a,1}} = 1, e'_{f_{a,2}} = 4)$ and $R'_a = R_a$. There are six possible orderings: $z^1 = a(1), a(2), b(1), b(2)$, $z^2 = a(1), b(1), a(2), b(2)$, $z^3 = a(1), b(1)b(2), a(2)$, $z^4 = b(1), a(1), a(2), b(2)$, $z^5 = b(1), a(1), b(2), a(2)$, $z^6 = b(1), b(2), a(1), a(2)$. It is easy to check that each of these orderings realizes with probability $\frac{1}{6}$. Note that $S^{uc} = \{s_3, s_4\}$ at I but $S^{uc} = \emptyset$ at $I_{\to_{e^a,R^a}}$. In $\phi^M(I)$, $b(1)$ represents $f_{b,2}$ and $b(2)$ represents $f_{b,1}$; in $\phi^M(I_{\to_{e^a,R^a}})$, $b(1)$ represents $f_{b,1}$ and $b(2)$ represents $f_{b,2}$. In both instances, $a(1)$ represents $f_{a,1}$ and $a(2)$ represents $f_{a,2}$. Observe that $\Pi_a^1 = \Pi_a^{\Phi^{\phi^{z^k}(I)}(a),\star}$

for $k = 1,2,3$; $\Pi_a^2 = \Pi_a^{\Phi^{\phi^{z^k}(I)}(a),\star}$ for $k = 4,5,6$; in addition, $\Pi_a^3 = \Pi_a^{\Phi^{\phi^{z^1}(I_{\to e^a,R^a})}(a),\star}$; $\Pi_a^4 = \Pi_a^{\Phi^{\phi^{z^k}(I_{\to e^a,R^a})}(a),\star}$ for $k = 2,3,4,5$; $\Pi_a^5 = \Pi_a^{\Phi^{\phi^{z^6}(I_{\to e^a,R^a})}(a),\star}$.

$f_{a,1}$'s expected delay given $\phi_a^{M\star}(I)$ is $\frac{1}{2}$ and given $\phi_a^{M\star}(I_{\to e^a,R^a})$ is $\frac{1}{6}$, so ϕ^M is not strategy-proof. The expected delay of $f_{a,2}$ given $\phi_a^{M\star}(I)$ is 0 and given $\phi_a^{M\star}(I_{\to e^a,R^a})$ is $\frac{5}{6}$.

Example D.6 and D.7:

For any instance I, let $I_{\to S',F'_\dagger,\Phi'_\dagger,n'}$ denote the instance that is the same as I except with $S, F_\dagger, \Phi_\dagger$ and n replaced by $S', F'_\dagger, \Phi'_\dagger$ and n', respectively. Let $F_\dagger^{a-f}$ be the resultant profile after F_a in $F_\dagger$ is replaced by $F_a \setminus \{f\}$. If airline a freezes $f \in F_a$ in a slot $s \in S_a$, the new instance is $I_{\to S\setminus\{s\},F_\dagger^{a-f},\Phi_\dagger^{a-s},n^{a-1}}$.[10]

Example D.6:

F	$f_{a,1}$	$f_{a,2}$	$f_{b,1}$
e	1	1	1
R	1	2	1
$\widehat{e}$		1	1
$\widehat{R}$		1	1

S	s_1	s_2	s_3
Π^1	$f_{a,1}$	$f_{a,2}$	$f_{b,1}$
Π^2	$f_{a,1}$	$f_{b,1}$	$f_{a,2}$

Suppose ϕ^M is employed and $S_a = \{s_1\}$. There are three possible orderings: $z^1 = a(1), a(2), b(1)$, $z^2 = a(1), b(1), a(2)$, and $z^3 = b(1), a(1), a(2)$. Each ordering realizes with probability $\frac{1}{3}$. $a(1)$ represents $f_{a,1}$ and $a(2)$ represents $f_{a,2}$. $f_{a,1}$ always get s_1. Observe that $f_{a,2}$ gets s_2 given z^1 and gets s_3 otherwise, so the expected delay of $f_{a,2}$ given $\phi_a^{M\star}(I)$ is $\frac{2}{3}$.

If a freezes $f_{a,1}$ in s_1, the resultant instance is $I' = I_{\to S\setminus\{s_1\},F_\dagger^{a-f_{a,1}},\Phi_\dagger^{a-s_1},n^{a-1}}$. There are two possible orderings: $z^4 = a(1), b(1)$ and $z^5 = b(1), a(1)$. Each ordering realizes with probability $\frac{1}{2}$. Now $a(1)$ represents $f_{a,2}$. Observe that $f_{a,2}$ gets s_2 given z^4 and gets s_3 given z^5, so the expected delay of $f_{a,2}$ given $\phi_a^{M\star}(I')$ is $\frac{1}{2}$.

Example D.7:

F	$f_{a,1}$	$f_{a,2}$	$f_{b,1}$
e	3	1	1
R	1	2	1
$\widehat{e}$		1	1
$\widehat{R}$		1	1

S	s_1	s_2	s_3
Π^1	$f_{a,2}$	$f_{b,1}$	$f_{a,1}$
Π^2	$f_{b,1}$	$f_{a,2}$	$f_{a,1}$

Suppose ϕ^M is employed and $S_a = \{s_3\}$. There are three possible orderings: $z^1 = a(1), a(2), b(1)$, $z^2 = a(1), b(1), a(2)$, and $z^3 = b(1), a(1), a(2)$. Each ordering realizes with probability $\frac{1}{3}$. $a(1)$ represents $f_{a,2}$ and $a(2)$ represents $f_{a,1}$. $f_{a,1}$ always get s_3. Observe that

[10] To keep our notation simple, we do not replace e.

$f_{a,2}$ gets s_2 given z^1 or z^2 and gets s_3 given z^3, so the expected delay of $f_{a,2}$ given $\phi_a^{M\star}(I)$ is $\frac{1}{3}$.

If a freezes $f_{a,1}$ in s_3, the resultant instance is $I' = I_{\to S\setminus\{s_3\}, F_\dagger^{a-f_{a,1}}, \Phi_\dagger^{a-s_3}, n^{a-1}}$. There are two possible orderings: $z^4 = a(1), b(1)$ and $z^5 = b(1), a(1)$. Each ordering realizes with probability $\frac{1}{2}$. Now $a(1)$ represents $f_{a,2}$. Observe that $f_{a,2}$ gets s_2 given z^4 and gets s_3 given z^5, so the expected delay of $f_{a,2}$ given $\phi_a^{M\star}(I)$ is $\frac{1}{2}$.

In both examples, $f = f_{a,1}$ is the most important flight of a, and the earliest feasible available slot for f, s, is in S_a. If s is a contested slot, then a might be better off by freezing f in s as demonstrated in Example D.6. The reason is that putting s and f into the instance would make a "pay" the position of $a(1)$ in z to get s. By contrast, if s is an uncontested slot, then freezing f in s might make a worse off, as demonstrated in Example D.7.

D.2 The Supplementary Algorithm

We suggest a supplementary algorithm for ϕ^M. Suppose $z^\star$ and z realized in the random ordering algorithm, and $\phi^z(I)$ is the realized landing schedule of ϕ^M. The induced slot ownership function of $\phi^z(I)$ is $\Phi^{\phi^z(I)}$. The following algorithm amends $\Phi^{\phi^z(I)}$ by assigning an additional M_a slots to each $a \in A$. Denote the resultant slot ownership function by $\Phi^{\phi^{z^\star}(I)}$.

For each a, eliminate each $a(i)$ with $i \leq |F_a|$ from $z^\star$ and denote the resultant ordering by $\mathfrak{z}^1$. Let $V_1 = S \setminus S_1$. For $t \in \{1, 2, ...\}$, repeat the following: Find s, which is the earliest slot in $V_t \cap S_A$ that satisfies the following requirement: $s \in S_a$ for some a and a has a surrogate in $\mathfrak{z}^t$. Assign the slot to a and remove the last surrogate of a from $\mathfrak{z}^t$. Update $\mathfrak{z}^t$ to $\mathfrak{z}^{t+1}$ and V^t to V^{t+1}. Stop if no slot satisfies the above requirement. If there is no remaining surrogate, stop; otherwise, denote the resultant ordering by $\mathfrak{z}^T$ and assign the earliest unassigned slots to the airlines sequentially according to $\mathfrak{z}^T$.

Example D.3 (continued):

Suppose $S_a = \{s_4\}$ and $f_{a,3}$ is another canceled flight of a. Suppose $z^\star = a(1), a(2), b(1), a(3)$. $\mathfrak{z}^1 = a(2), a(3)$. $V_1 = \{s_1, s_4, ...\}$. The earliest slot in $V_1 \cap S_A$ that satisfies the requirement is s_4, so s_4 is assigned to a and $a(3)$ is removed from $\mathfrak{z}^1$. $\mathfrak{z}^2 = a(2)$ and $V_2 = \{s_1\}$. $V_2 \cap S_A = \emptyset$, so $\mathfrak{z}^T = a(2)$. s_1 is assigned to a.

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